Joint Decision-Making of Order Policies and Advertising Cost for the Retailer with Loss-aversion

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Abstract: This paper deals with the joint decision-making problem of advertising cost and order quantity for the retailer with loss-aversion under uncertain demand, and the corresponding joint decision-making model is built based on the loss-aversion utility function within prospect theory framework. The results indicate that both the optimal order quantity and advertising cost of the retailer with loss-aversion are less than those of the risk neutral retailer, the theoretical explanation can be given to interpret the bias found in the empirical study. Through theoretical analysis, this paper also studies the influence of model parameters on the optimal order quantity and advertising cost.

Keywords: loss-aversion; advertising cost; order policy

I. Introduction

With the high speed development of technology and continuous growth of consumers' personalized demands, the product, as a result, has its update rate keep increasing and its life span keep shortening, wherefore more and more products take on the characteristics of fashion or seasonal goods. Generally, such product takes a long period of production and a short period of sale, which easily brings about the Mismatch Phenomena between supply and demand in practice. For instance, computers branded as Value Point of IBM boasted a surplus stock of more than 7 hundred million dollars in 1994[1], but in 1995, their computers branded as Aptiva suffered a loss of potential earnings about more than a hundred million dollars just due to short supply[2]. Compared with ordinary products, fashion or seasonal goods face a bigger challenge on inventory control, which makes the stock decision problem of fashion or seasonal goods become a hot topic for so many investigators and decision makers in recent years.

The basic model in the research of the stock decision problem of fashion or seasonal goods is the classic newsvendor model. However, in order to better simulate practical business environment, many scholars make further efforts on the development of the classic newsvendor model. Khoujahas made a relatively comprehensive overview of newsvendor model before 21st century [3]. In the new century, the newsvendor model is still evolving. For instance, Carr and Lovejoy studied Inverse Newsvendor Problem [4]; Van Mieghem and Rudi developed Newsvendor Network Model by allowing multiple products coexist with multi-processing and multi-stocking[5]; Berk studied the

newsvendor problem concerning the update of required information, etc[6]. Regretfully, all of these researches failed to study the influence of non-price factors like advertisement on product demand, whereas advertisement was an important method which practice managers used to increase product demand. Supposing that the expected demand function was a power function of advertising cost, Khouja and Robbins respectively set the maximization of expected profit and the maximization of the probability of a certain profit level as their targets, and probed into the joint decision-making problem of retailers' optimal order quantity and advertising cost[7]; Zhou Yongwu and Yang Shanlin supposing expected demand was a generic function of advertising cost, established a joint decision-making model even more general than the one Khouja and Robbins put forth[8]; Bu Xiangzhi and his colleagues took into account the influence of advertising investment on demand, and established a model where multiple enterprises gaming with each other in order quantity and advertising investment. Cao Xiyu and her colleagues took into account the influence of joint investment of both brand advertising and local promotion advertising on demand, and established a model where joint investment of brand advertising and local promotion advertising gaming with order quantity when demand was not stable [9]; Wang Junping and her colleagues, in the precondition that expected demand was a generic function of selling price and advertising cost, generalized the traditional newsvendor model [10].

However, all those research results are based on an assumption that retailers are risk neutral that is to say, retailers are always seeking to maximize expected profit. Nevertheless, some empirical researches find that in practice retailers do not stick to the principle of profit maximization when they are making decisions about orders. Schweitzer and Cachon made a study on order decision problem of loss-averse retailers with the help of classic newsvendor model. They used a piecewise linear loss-aversion utility function. Their research results indicated that the optimal order quantity of loss-averse retailers was not only less than the quantity introduced by classic newsvendor model, but also decreasing with the increase of loss-aversion level [11]. Chinese scholar Wen Ping made a research on loss-averse newsvendor model within the prospect theory framework, and obtained the optimal solution of newsvendor model [12]. However, all these researches are not suitable to S-type utility function. Recently, Zhao and his colleagues described decision makers' behavior characteristics by means of S-type

exponential utility function; they found the optimal order quantity of loss-averse retailers was less than that of classic newsvendor model based on numerical analysis [13]. But, these researches ignored the influence of advertising on demand.

This paper assumes that expected demand is a generic function of advertising cost, and the joint decision-making model combining the order quantity and advertising cost of the retailer with loss-aversion is proposed, the analysis method for the optimal order quantity and advertising cost is put up through the model, and finally the influence of the extent of loss-aversion, the retail price and the purchase cost on the optimal order quantity and advertising cost are analyzed theoretically.

II. Model Description and Symbol Hypothesis

The symbols will be utilized by this paper are defined as follows: q, the order quantity of retailer at the beginning of sale season, (the decision variable); u, the advertising cost is used by the retailer at the beginning of sale season (decision variables); p', the retail price per unit of product; $D(u)$,the expected demand of market; v', the salvage value of the unsold product; c', the purchase cost per unit of product. We assume that demand of the retailer faced is random, which is a function of the advertising cost, that is $D(\varepsilon,u)= D(u)\cdot\varepsilon$. Where ε is a positive random variable that its mean is 1, its probability distribution function and probability density function are $F(\cdot)$ and $f(\cdot)(f(x)>0)$ accordingly. In addition, according to the basic knowledge of marketing, we can assume that $D'(u)> 0$ and $D''(u) <0$, and because the number of potential customers is limited, the further assumption $\lim_{u\to\infty} D(u) = D_m$, can be set ,in which D_m is the most expected demand of customers for this product.

Based on the above assumptions, the profit function of retailer can be expressed as:

$$\pi(q,u)=\begin{cases} p'D(\varepsilon,u)-c'q+v'[q-D(\varepsilon,u)]-u, & D(\varepsilon,u)\le q, \\ (p'-v')q-(c'-v')q-u, & D(\varepsilon,u)> q. \end{cases}$$

By assumptions of $p = p'-v'$ and $c = c'-v'$, Eq.(1) can be simplified as:

$$\pi(q,u)=\begin{cases} pD(\varepsilon,u)-cq-u, & D(\varepsilon,u)\le q, \\ (p-c)q-u, & D(\varepsilon,u)> q. \end{cases} \quad (2)$$

By Eq.(2), it is easy to known that there is a break-even demand point $q(q,u)=\dfrac{u+cq}{p}$,for the retailer(Under the normal condition, the advertising cost input by the retailer will not exceed the profits obtained in the certain demand .Hence we draw $0\le u\le (p-c)q$, from which can guarantee $0< q(q,u)\le q$.). When the demand of product is just equal to $q_1(q,u)$, the returns of retailer is 0

and the retailer makes no compensation; and when the demand is less than $q_1(q,u)$, the retailer profit is negative and the retailer is facing a loss; and when the demand is greater than $q_1(q,u)$, the retailer's profit is positive.

By Eq.(2), we can conclude the expected profit function for the retailer as:

$$E\left[\pi(q,u)\right]=\int_0^{\frac{q}{D(u)}} pD(u)xf(x)dx+\int_{\frac{q}{D(u)}}^{\infty} pqf(x)dx-cq-u$$

$$(3)$$

We set the constant W_0 as the initial wealth for the retailer ,then the form of loss-aversion utility function could be considered as (Wang and Webster[13], Schweitzer and Cachon[11]):

$$U(W)=\begin{cases} W-W_0, & W\ge W_0, \\ \lambda(W-W_0), & W< W_0, \end{cases} \quad (4)$$

λ $(\ge 1)$is the measurement parameter for the retailer with different extent of loss-aversion. When $\lambda=1$, the retailer is risk neutral; and when $\lambda>1$, the retailer is endowed with the character of loss-aversion, and the larger λ the higher extent of loss-aversion. From the functional form of $U(W)$ we know, the utility function is a piecewise linear function with initial wealth reference point W_0 as the cutoff point, we can assume $W_0=0$ in order to facilitate the discussion and without loss of generality.

Thus, according to Eq.(2) and Eq.(4) we get the expected utility function of the retailer loss-aversion as:

$$E\left[U(\pi(q,u))\right]=E\left[\pi(q,u)\right]+(\lambda-1)(\int_0^{\frac{q(q,u)}{D(u)}}\left[pxD(u)-cq-u\right]f(x)dx)$$

$$(5)$$

Eq.(5) has the following economics meaning: the expected utility of the retailer with loss-aversion is the sum of the expected profit of the retailer with risk-neutral and the psychological loss ($\lambda \ge 1$)caused by insufficient ordering .

III. The Optimal Order Quantity and Advertising Cost for the Retailer with Loss-Aversion

The retailer with loss-aversion should choose the optimal order quantity and advertising cost (q,u) on the principle maximization of the expected utility $E\left[U(\pi(q,u))\right]$. Thus, we get the first-order partial derivatives with regard to q and u and seek to make them equal to 0 according to the existence of the necessary conditions for an extremum:

$$F(q,u)=\frac{\partial E\left[U(\pi(q,u))\right]}{\partial q}=(p-c)\overline{F}(\frac{q}{D(u)})$$

$$-cF(\frac{q}{D(u)})-(\lambda-1)cF(\frac{q(q,u)}{D(u)})=0 \quad (6)$$

$$G(q,u) = \frac{\partial E[U(\pi(q,u))]}{\partial u} = \int_0^{\frac{q}{D(u)}} pD'(u)xf(x)dx - 1$$

$$+ (\lambda - 1)(\int_0^{\frac{q(q,u)}{D(u)}} [pD'(u)x - 1]f(x)dx) = 0 \ (7)$$

For further discussion about property of the optimal order quantity and advertising cost, the following theorem 1is given to prove the existence of the optimal order quantity and advertising cost.

Theorem 1 the Hessian matrix of the expected utility function $E[U(\pi(q,u))]$ is negative definite, and thus there is one and only one optimal order quantity and advertising cost $(q_\lambda^*, u_\lambda^*)$ to maximize the expected utility of the retailer with loss-aversion, and the optimal solutions satisfy Eq. (6) and Eq.(7)(proof in Appendix 1).

From Theorem 1, we consider the special condition $\lambda \to 1$ and get the following Lemma.

Lemma when $\lambda \to 1$, the optimal order quantity and advertising cost are the same as the results of Yong-Wu Zhou and Shan-Lin Yang (2002) [8], that is , the optimal order quantity q_1^* satisfies $q_1^* = D(u_1^*)F^{-1}(\xi)$ and the optimal advertising cost u_1^* satisfies

$$pD'(u_1^*)\int_0^{F^{-1}(\xi)} xf(x)dx - 1 = 0 \text{ ,where } \xi = (p-c)/p \ .$$

The following discussion without considering the shortage cost focuses on the impact of the extent of loss-aversion, the retail price and the purchase cost on the optimal strategies.

IV. The Sensitivity Analysis

The impact of the retailer's loss-aversion extent on the optimal strategy

The impact of the retailer's loss-aversion extent on the optimal strategy will be illustrated by the following Theorem 2, Theorem 2 will show that both the optimal order quantity and advertising cost of the retailer with loss-aversion will be lower than the ones with the risk-neutral retailer (profit maximization) under certain conditions.(proof in Appendix 2)

Theorem 2 the optimal advertising cost u_λ^* of the retailer with loss-aversion will be always lower than the optimal advertising cost u_1^* of the risk-neutral retailer under the same expected demand; the optimal order quantity q_λ^* of the retailer with loss-aversion will be always lower than the order quantity q_1^* of the risk-neutral retailer under the same expected demand.

In order to prove theorem 2, the following two corollaries are given.

Corollary 1 when $D(u_\lambda^*) - (u_\lambda^* + cq_\lambda^*)D'(u_\lambda^*) < 0$, the more advertising cost is spent by the retailer with loss-aversion, then the more product quantity is order by the retailer, that is $\frac{dq_\lambda^*}{du_\lambda^*} > 0$.(proof in Appendix 3)

Corollary 2 λ is more larger, the optimal advertising cost is more lower, that is $\frac{du_\lambda^*}{d\lambda} < 0$. (Proof in Appendix 4)

The impact of the retailer's purchase cost and retail price on the optimal strategy

Based on the above analysis, the optimal order quantity and advertising cost of the retailer with the loss-aversion coefficient λ should satisfy the Eq. (9) and Eq.(10), from which we know that the optimal order quantity and advertising cost are not only related to the extent of loss-aversion, but also are related to the retail price p and the purchase price c.

The following Theorem 3 and Theorem 4 will show the impact of the retail price p and the purchase price c on the optimal advertising cost, and Corollary 3 and the Corollary 4 will show the impact of the retail price p and the purchase price c on the optimal order quantity.

Theorem 3 when $D(u_\lambda^*) - (u_\lambda^* + cq_\lambda^*)D'(u_\lambda^*) < 0$ and the salvage value stays the same in the end of sales, the higher the purchase cost is, the lower the advertising cost the retailer with loss-aversion will spend, that is $\frac{du_\lambda^*}{dc'} < 0$.(proof in Appendix 5)

Corollary 3 when $D(u_\lambda^*) - (u_\lambda^* + cq_\lambda^*)D'(u_\lambda^*) < 0$ and the salvage value stays the same in the end of sales, the higher the purchase cost is, the lower the product quantity the retailer with loss-aversion will order, that is $\frac{dq_\lambda^*}{dc'} < 0$.(proof in Appendix 5)

Theorem 4 when $D(u_\lambda^*) - (u_\lambda^* + cq_\lambda^*)D'(u_\lambda^*) < 0$ and the salvage value stays the same in the end of sales, the higher the retail price is, the higher the advertisement cost the retailer with loss-aversion will spend, that is $\frac{du_\lambda^*}{dp'} > 0$.(proof in Appendix 6)

Corollary 4 when $D(u_\lambda^*) - (u_\lambda^* + cq_\lambda^*)D'(u_\lambda^*) < 0$ and the salvage value stays the same in the end of sales, the higher the retail price is, the higher the product quantity the retailer with loss-aversion will order, that is $\frac{dq_\lambda^*}{dp'} > 0$.(proof in Appendix 6)

V. Conclusion

This paper studies the joint decision-making problem of the order quantity and advertising without considering the shortage cost, and then the sensitivity analysis of the loss-aversion coefficient, the product purchase cost and the retail price are discussed, the results indicate that the optimal order quantity and advertising cost will increase with the retail price and decrease with the purchase cost and the extent of loss-aversion. The further expansion of this model is that two-echelon supply chain coordination strategies with the loss-aversion retailer and demand influenced by the advertisement.

Appdenices
Please contact the authors for the appendices if necessary.

Acknowledgement

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